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14. 1 per cent. = $\frac{1}{100}$ of \$7,500 = \$60.

15. 100 per cent. = \$6000 = the cost of the house.

Also solved by COOPER D. SCHMITT, F. R. HONEY, M. E. GRABER, EDW. R. ROBBINS, B. F. SINE, J. SCHEFFER, J. F. TRAVIS, EARL R. GIBSON, FREMONT CRANE, HENRY HEATON, J. H. DRUMMOND, and G. B. M. ZERR.

Solved in a similar manner, though more briefly, by Walter H. Drane. Professor Drane has some doubts as to whether the above is strictly an arithmetical solution and asks the question, What is the exact difference between an arithmetical and an algebraic solution? This question I have answered in the last number of the MONTHLY. The above is not an arithmetical solution at all. Professor Ellwood's last solution is algebraic. My solution of problem 93, aside from the principles borrowed from geometry, is purely arithmetical, because all the operations are performed at once upon certain numbers and the results of these operations summed give the final result. The selling price is printed in italics for sake of distinction. [ED. F.]

ALGEBRA.

83. Proposed by J. MARCUS BOORMAN, Consultative Mechanician and Counselor at Law, Woodmers, Long Island, New York.

Solve $x^2 + y = 8 \dots (I)$; $y^2 + x = 60 \dots (II)$, true to four decimals.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

(I) in (II) gives $x^4 - 16x^2 + x = -4$.

$\therefore x = -4, y = -8. \therefore x = 3.935432, y = -7.487627$;

$x = .537401, y = 7.711199$;

$x = -.472833, y = 7.776428$.

Solved similarly by J. H. DRUMMOND, and EDW. R. ROBBINS.

II. Solution by the PROPOSER.

[N. B. Change $\pm_1 \dots \mp_1$ only in unison, — thus $S = \pm_1(x + x_1)$ to $\Sigma = \pm_1(y + y_1)$; $S_1 = (x_1 + x_{11})$ to $\Sigma_1 = (y_1 + y_{11})$; $S_a = (x + x_{11})$ to $\Sigma_a = \pm_1(y + y_{11})$, 12 roots.]

The "Empirical" gives (E)

$S^4 - 2[S \pm_1 \sqrt{(8^2 - 60)}]S^2 + Id^2 = Id^2 \pm_1 \frac{1}{4}f_1/[1/(8^2 - 60)]$. [$f = 1$, factor of x in (I); $S = \text{sum } x + x_1$; $\Sigma = \text{co-sum } (y + y_1)$, etc. *Analyst* for 1882, IX, page 122].

$f = f^2 = 1 \dots \therefore$

(III) $S^2 = 10 \pm_1 \sqrt{100 \frac{1}{2}} = 10 \pm 10.0062484 \dots \therefore S = \pm 4.472834, 2 +$,

(IV) $S_1^2 = 6 \pm_1 \sqrt{35 \frac{1}{2}} = 6 \pm 5.9895761$. True $S = \underline{4.472833, 9 +}$.

Error 0.000000, 3.

By (IV) $S_1 = \pm 3.462596, 5$, — its reciprocal is $\Sigma_1 = 0.288800, 4 +$,

True $S_1 = 3.462598, 42$. Reciprocal of S is $\Sigma = 0.223571, 9 +$,

Error 0.000001, 9 $\therefore S_a = 0.064567, 6$; $\Sigma_a = 15.487627, 6 +$.

[Thus reducing the "ir-reducible" case]. Q. V. D.

By $\Sigma = f(1/S)$, $f = 1 = S\Sigma = SS_1S_a$. Any S or Σ will give, by

(A) $x = \pm_1 \frac{1}{2}S \pm_1 \sqrt{[8 - \frac{1}{2}S^2 \mp_1 \frac{1}{2}f(1/S)]}$. ($f = f^2 = 1$ here.)

(B) $y = \pm_1 \frac{1}{2}\Sigma \pm_1 \sqrt{[60 - \frac{1}{2}\Sigma^2 \mp_1 \frac{1}{2}f^2(1/\Sigma)]}$, the whole eight roots, viz.:

$x = 0.537401 +$, $x_1 = 3.935432$, $x_{11} = -0.472833, 9$, $x_a = -4$;

$y = 7.711199 +$, $y_1 = -7.487627$, $y_{11} = 7.776428, 1$, $y_a = -8$.

Compare $x_1 = 3.935432 \quad 331970 \quad 029801 \quad 953731,229 +$

$$x_a = -4.$$

$$S_a = \mp 0.064568 \quad 668029 \quad 970198 \quad 946268,770 \mp$$

$$y_1 = -7.487627 \quad 639515 \quad 066744 \quad 071631,521 +$$

$$y_a = 8.$$

$$(\Sigma)_a = \pm 0.512372 \quad 360484 \quad 933255 \quad 928368,478 -$$

$$x = 0.537401 \quad 577025 \quad 225760 \quad 614153,4040 +$$

$$x_2 = -0.472833 \quad 908995 \quad 255561 \quad 667884,633 +$$

$$S_a = -0.064568 \quad 668029 \quad 970198 \quad 946268,770 \pm$$

$$\text{Also, } y = 7.711199 \quad 545010 \quad 300343 \quad 929222,6193 +;$$

$$y_2 = 7.775428 \quad 094504 \quad 266400 \quad 142408,902 \pm.$$

S , Σ , etc., is stated $\pm$ because the remaining two roots of x or y so make it. Note that $S_a = -S_a$, and that the two determine the final figure 0! Also observe the curious recurrences 777 twice, 222, 111, etc.

The "Empirical" is one of three novelties found in 1875-6. Des Cartes' $S^6 - 4aS^4 + 4bS^2 - f^2 = 0 \dots (C)$, so stated factors into $S^3 \mp 2xS^2 - 2yS \pm f = 0!$ Also to $S^3 - 2[a \pm 1](a^2 - b)]S \mp \eta_1 [1/(a^2 - b)](1/S) = 0 \dots (D)!$

Change $\eta_1 [1/(a^2 - b)](1/S)$ "empirically" into $\frac{1}{2}f_1/[1(a^2 - b)](1/S)$ gives (E) above. For, say twelve decimals true correct $\frac{1}{2}f$, etc., into $(\frac{1}{2}+)v$, etc., by $v + (\frac{1}{2}+)v^2 [1/(a^2 - b)](1/S^2) = f$.

NOTE. Josiah H. Drummond remarks of the Proposer's solution of No. 78: It seems to me that his reasoning is faulty, and his conclusion erroneous. His reasoning seems to be: If $x^2 = 36$ (1) it follows that $x = 1296$ (2) and as (2) has four roots, therefore (1) has four roots. Again ± 2 and ∓ 2 , taken by themselves, are precisely the same; the first is ± 2 or -2 , and the second is precisely the same, save in the order of writing or reading them, which does not affect the mathematical result."

GEOMETRY.

91. Proposed by LEONARD E. DICKSON, Ph. D., Instructor in Mathematics in the University of California, Berkeley, Cal.

If a point A remain fixed while a point B moves along a given straight line, prove that the locus of the vertex C of the triangle ABC , similar to a given triangle and lying always on the same side of AB , is a straight line. Verify geometrically for the case in which the angles at A and C remain equal.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let MN be the fixed line, denote AO , which is constant, by h , and the co-ordinates OD by x and CD by y , and extend AC to E , denote $\angle AEB$ by E , and the constant angles of triangle ABC by A , B , C . We have

$$AB = \frac{h}{\sin(A+E)}, \quad BC = \frac{y}{\sin(C-E)}:$$

$$\therefore \frac{h}{\sin(A+E)} : \frac{y}{\sin(C-E)} = \sin C : \sin A,$$

